[12.13] Poincare’s Lemma for *p* = 1 in **ℝ2**. Let ** be a 1-form such that *d* = 0. Show that there is a scalar field  : **ℝ2 → ℝ**such that locally * =* d.

(0,0)

(x,0)

(x,y)

l1

l 2

From problem [12.11],



Without loss of generality, let’s choose our local point to be (0,0), and assume the point (x,y) is in an open connected neighborhood of (0,0) so that we can joint them with the lines l1 and l2 as shown.

Define  That is, we integrate from (0,0) to (x,y) using *A* along l1 and *B* along l2.

Restricted to l2, is a function of just *y*. Let *b*(*y*) be the antiderivative of  That is,  So,



Restricted to l1, is a function of just *x*. Let *a*(*x*) be the antiderivative of  That is,  Similarly, restricted to l2,  is a function of just t. So,



Finally, we have

